Migration of a Large Gas-Bubble under the Lack of Gravity in a Rotating Liquid

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THE availability of large boosters, which may transport men and material into an Earth orbit, confronts us more intensively with the new phenomenon of zero-gravity. Previously, the state of weightlessness could only be accomplished for a few seconds, which in many cases, in spite of careful experimental preparations, were not sufficient for a system to reach its steady-state condition. Through the use of Saturn IB, Saturn V and the planned space shuttle, one is now in the position to put a workshop in orbit for the purpose of manufacturing under the lack of gravity. The frequent use of such a shuttle fleet makes it possible to provide people and material to the workshop. This provides us with an environment that is not possible on Earth, because zero-gravity is combined with an unlimited vacuum and low temperatures. These are the basis for new manufacturing processes and new technologies, which are very difficult and expensive or even impossible in an earthbound workshop. The lack of gravity would enable us to cast perfect ball bearings, perfect optical lenses and combine metals of various kinds, etc. In the casting of metal and glass it is mandatory to degas the material. On Earth this is accomplished by the bouyancy forces, which transport the gas bubble towards the free surface of the liquified material. In an orbital workshop, however, these forces are due to the lack of gravity not present, and the gas bubble rests wherever it may be located. This, of course, is a serious problem for the casting of a proper material. Therefore the question arises of how one could transport the gas bubbles to a location where they could be taken out of the material. This could be accomplished by an acceleration field, i.e., by spinning up the material, which moves all gas bubbles towards the axis of rotation. For the degassing process of a material, the time required for a bubble to move under the action of the centrifugal force from a certain location to the axis of rotation is of great importance. The following Note treats the migration of a large (volume $V > 0.3$ in.^{3}) gas bubble, where the effect of surface tension is considered negligible and for which the shape becomes flat at the rear face of the bubble. In this case the bubble is presented as a spherial cap and its motion is approximately determined. Although the rear face of the bubble is rather unsteady, and the edges of the spherical cap move irregularly, the whole front appears quite steady and of spherical form. These features of the gas bubble, which is now considered to be in an infinite liquid media, allow the derivation of a simple formula for the motion of it in a rotating liquid.

Let us consider the steady flow near the stagnation point of the bubble (Fig. 1). The equation of motion (Euler's equation) for a rotating system in zero gravity is given by

$$\tfrac{1}{2}\cdot\text{grad}\,\mathbf{v}^2 - [\mathbf{v}\times\text{curl}\mathbf{v}] + (1/\varsigma)\,\text{grad}\rho + \{\mathbf{\Omega}\times[\mathbf{\Omega}\times\mathbf{r}]\} + 2[\mathbf{\Omega}\times\mathbf{v}] = 0 \quad (1)$$

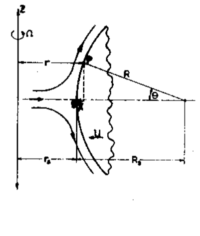

Fig. 1 Geometry of bubble.

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where $\mathbf{\Omega} = \Omega\mathbf{k}$ and $\mathbf{r} = r\mathbf{e}_r + z\mathbf{k}$. $\mathbf{\Omega}$ is the steady angular velocity. The last term represents the Coriolis force, whereas the term in front of it is the centrifugal force. The Coriolis force has zero component in the direction of $\mathbf{v}$, and the centrifugal force is then

$$\{\mathbf{\Omega}\times[\mathbf{\Omega}\times\mathbf{r}]\} = -\tfrac{1}{2}\,\text{grad}[\mathbf{\Omega}\times\mathbf{r}]^2$$

which yields after integration of the above Eq. (1) along the streamline, the Bernoulli equation:

$$v^2/2 + p/p - (\Omega^2/2)r^2 = \text{const} \quad (2)$$

Employing now Bernoulli's equation for the streamline at the bubble surface, yields, since the pressure must be uniform over the forward face of the bubble, the expression

$$v^2 = \Omega^2(r^2 - r_s{}^2) \quad (3)$$

where the subscript s refers to the stagnation point. It is

$$r = r_s + X \qquad X = R_s - R\cos\theta \quad (4)$$

R_s and R are the radii of curvature at the stagnation point and the point P on the forward face of the bubble, respectively. The values R and θ are spherical coordinates with their origin at the center of curvature of the bubble forward face. The speed of the liquid at the bubble surface is v; it depends upon the speed with which the bubble travels through the liquid. It also must depend to a certain extent on the bubble size and geometric shape. In the neighborhood of the stagnation point, the velocity v varies linearly with the distance from S and may therefore be written as

$$v = \delta U\theta \quad (5)$$

where δ is a dimensionless constant depending only on the geometric shape of the bubble. Since the geometric shape of the bubble is very closely that of a spherical cap, the value $R \approx R_s$ and Bernoulli's equation yields

$$\delta^2U^2\theta^2 = \theta^2\Omega^2r_sR$$

This has been obtained by expanding $\cos\theta \approx 1 - \theta^2/2$ and neglecting terms of order $0(\theta^3)$ and higher. Therefore,

$$\delta^2U^2 = \Omega^2r_sR \quad (6)$$

The fact that the boundary of the region of irrotational flow on the foreward face of the bubble remains nearly spherical gives us a means for the determination of the geometrical shape factor δ. The velocity v at P of a sphere

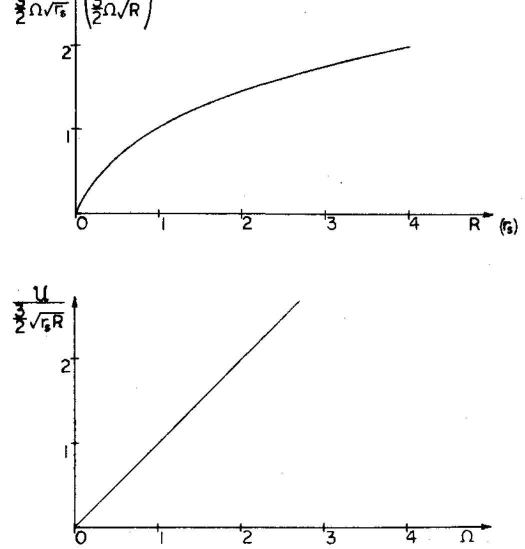

Fig. 2 Dependency of speed of migration U of large bubble.

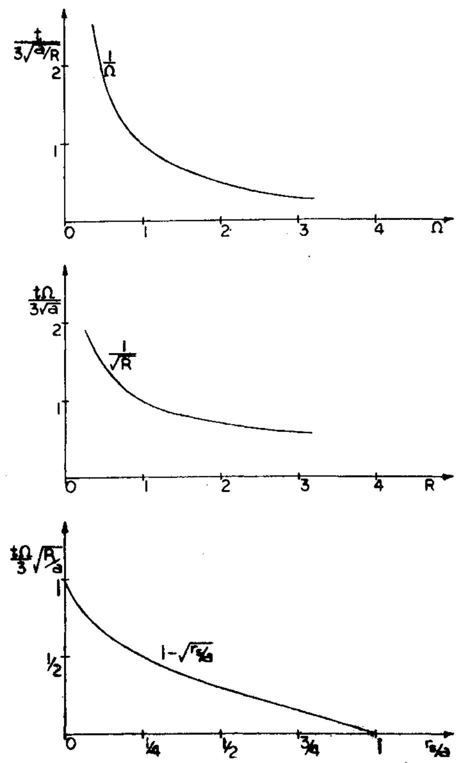

Fig. 3 Time of migration of bubble.

moving with the velocity U through a nonviscous liquid is given by potential theory as

$$v = \tfrac{3}{2}U\sin\theta \approx \tfrac{3}{2}U\theta \tag{7}$$

With this, the speed of migration of the bubble under the action of centrifugal force and under the lack of gravity is obtained by comparison of Eqs. (5) and (7) to be ($\delta = \frac{3}{2}$)

$$U = \tfrac{2}{3}\Omega(r_sR)^{1/2} \tag{8}$$

It may be seen from this that the speed of migration of a large bubble is proportional to the speed of rotation Ω and proportional to the square root of the distance of the stagnation point of the bubble from the axis of rotation. We conclude furthermore that it is proportional to the square root of the radius of curvature of the bubble, indicating that larger bubbles migrate faster (Fig. 2). For a small distance of the bubble from the axis of rotation its speed of migration is slow.

To determine the time of migration involved in the motion of a bubble, the following equation has to be integrated:

$$dr_s/dt = -\tfrac{2}{3}\Omega(Rr_s)^{1/2} \tag{9}$$

Assuming that the rotational speed Ω is kept constant and that the radius of curvature remains constant throughout the motion of migration, Eq. (9) yields

$$r_s = a - \tfrac{2}{3}\Omega(Ra)^{1/2}\cdot t + (\Omega^2/9)Rt^2 \tag{10}$$

where the distance of the stagnation point of the bubble from the rotational axis at the time $t = 0$ is considered to be $r_s(0) = a$. The time of migration from a to a location of the stagnation point of the bubble r_s is then

$$t = [3/\Omega(R)^{1/2}][(a)^{1/2} - (r_s)^{1/2}] \tag{11}$$

It may be seen that the time elapsing for a bubble moving from $r_s = a$ to the axis of rotation $r_s = 0$ is given by

$$t = 3(a)^{1/2}/\Omega(R)^{1/2} \tag{12}$$

which expresses (Fig. 3) that it is indirectly proportional to the rotational speed, proportional to the square root of the distance of the stagnation point of the bubble from the axis of rotation and indirectly proportional to the square root of the radius of curvature of the bubble. The larger the rotational speed, the earlier the bubble will reach the axis of rotation. A bubble of large radius of curvature shall reach the axis of rotation earlier than a bubble of small radius of curvature.

Electron Number Density at Shock Front due to Precursor Photoionization

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DOBBINS has presented the theory of shock precursors as applied to shock tube experiments in Ref. 1. He assumed black-body emission in the primary continuum from equilibrium region behind the shock which is likely to be violated as the initial pressure is lowered. We worked out this case in Ref. 2 and present here the main developments.

A review of the literature on shock precursor ionization is given by Pirri and Clarke.[3] Weymann,[4] Holmes and Weymann[5] presented experimental evidence for electron diffusion and photoionization as causing the observed precursors in shock tubes. They found that in any case photoionization dominates the near precursor.

The profile of electron number density ahead of the shock due to photoionization of atoms in ground state is described by

$$d(n_eu)/d\xi = -n_aQ_{13} \tag{1}$$

with the rate coefficient Q_{13} given by

$$Q_{13} = 4\pi\int_{\nu_1}^{\infty}\psi_{13}J_\nu d\nu \tag{2}$$

The notation is the same as in Ref. 6. The bound-free absorption coefficient a_ν decreases with frequency and can be represented by

$$a_\nu = P/\mu^n, \qquad \mu = \nu/\nu_1, \qquad a_\nu = \psi_{13}h\nu \tag{3}$$

$h\nu_1$ is the ionization potential. In Eq. (3), P and n are constants which depend on the particular atomic gas considered. We show later that the electron number density at the shock front due to photoionization is independent of both P and n. J_ν is the average spectral intensity given by

$$J_\nu = \left(\frac{1}{4\pi}\right)\int I_\nu d\omega \tag{4}$$

The intensity at any point A on the axis (Fig. 1) is

$$I_\nu(\xi,\theta) = B_\nu[1 - \exp(-K_\nu^{(2)}L\sec\theta)] \times \exp(-K_\nu^{(1)}\xi\sec\theta) \tag{5}$$

where B_ν is the Planck's function at the downstream equilibrium T_2. We have assumed LTE in the shock-heated gas and

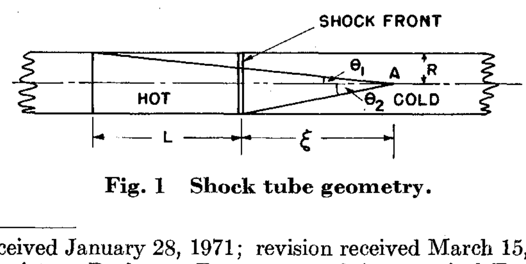

Fig. 1 Shock tube geometry.

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